

QCD corrections to the decay $H^+ \rightarrow t\bar{b}$ in the Minimal Supersymmetric Standard Model

A. Bartl,¹ H. Eberl,² K. Hidaka,³ T. Kon,⁴
 W. Majerotto², and Y. Yamada^{5*}

¹*Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria*

²*Institut für Hochenergiephysik der Österreichische Akademie der Wissenschaften, A-1050 Vienna, Austria*

³*Department of Physics, Tokyo Gakugei University, Koganei, Tokyo 184, Japan*

⁴*Faculty of Engineering, Seikei University, Musashino, Tokyo 180, Japan*

⁵*Theory Group, National Laboratory for High Energy Physics (KEK), Tsukuba, Ibaraki 305, Japan*

Abstract

We present a complete calculation of the $\mathcal{O}(\alpha_s)$ QCD corrections to the width of the decay $H^+ \rightarrow t\bar{b}$ within the Minimal Supersymmetric Standard Model. We find that the QCD corrections are quite important, and that the supersymmetric QCD corrections (due to gluino, \tilde{t} and \tilde{b} exchange) can be comparable to or even larger than the standard QCD corrections in a large region of the supersymmetric parameter space. This is mainly due to the effect of large left-right mixings of stop (\tilde{t}) and sbottom (\tilde{b}). This could significantly affect the phenomenology of the H^+ search.

*Present address: Physics Department, University of Wisconsin, Madison, WI 53706, USA

1 Introduction

The existence of a charged Higgs boson H^+ would be a clear indication that the Standard Model must be extended. For example, the Minimal Supersymmetric Standard Model (MSSM) [1] with two Higgs doublets predicts the existence of five physical Higgs bosons h^0, H^0, A^0 , and H^\pm [2, 3]. If all supersymmetric (SUSY) particles are heavy enough, H^+ decays dominantly into $t\bar{b}$ above the $t\bar{b}$ threshold [2, 4]. In refs. [5, 6] all decay modes of H^+ including the SUSY-particle modes were studied in the case that the SUSY-particles are relatively light: it was found that the $t\bar{b}$ mode remains important even in this case (though the $\tilde{t}\tilde{b}$ mode can be dominant in a wide range of the MSSM parameters). Thus it is important to calculate the QCD corrections to the $t\bar{b}$ mode as they could significantly affect the phenomenology of the H^+ search. The standard QCD corrections to the $t\bar{b}$ mode were already calculated [7]: they can be large (+10% to -50%). There also exist calculations of the SUSY-QCD corrections within the MSSM [8, 9]. However, in ref. [8] the squark-mixing was neglected. The calculation in ref. [9] is incomplete as the wave function and mass renormalizations were omitted. A calculation of SUSY-QCD corrections to the related process $t \rightarrow H^+ b$ was done recently in ref. [10].

In this paper we present a complete calculation of the $\mathcal{O}(\alpha_s)$ QCD corrections to the width of $H^+ \rightarrow t\bar{b}$ within the MSSM. We include the left-right mixings of both the $\tilde{t}_{L,R}$ squarks and the $\tilde{b}_{L,R}$ squarks. We adopt the on-shell renormalization scheme.

2 QCD one-loop contributions

The one-loop corrected amplitude of the decay $H^+(p) \rightarrow t(k_t)\bar{b}(k_{\bar{b}})$ ($p = k_t + k_{\bar{b}}$) can be written as

$$\mathcal{M} = i\bar{t}(Y_1 P_R + Y_2 P_L)b \quad (1)$$

with $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and the one-loop corrected couplings:

$$Y_i = y_i + \delta Y_i^{(g)} + \delta Y_i^{(\tilde{g})} \quad (i = 1, 2), \quad (2)$$

where y_i are the tree-level couplings corresponding to Fig. 1a:

$$\begin{aligned} y_1 &= \frac{g}{\sqrt{2}m_W} m_b \tan \beta = h_b \sin \beta, \\ y_2 &= \frac{g}{\sqrt{2}m_W} m_t \cot \beta = h_t \cos \beta, \end{aligned} \quad (3)$$

with g being the SU(2) coupling. $\delta Y_i^{(g)}$ and $\delta Y_i^{(\tilde{g})}$ are the contributions from gluon and gluino exchanges, respectively (as shown in Figs. 1b and 1c).

The tree-level decay width is given by:

$$\Gamma^{\text{tree}}(H^+ \rightarrow t\bar{b}) = \frac{N_C \kappa}{16\pi m_{H^+}^3} [(m_{H^+}^2 - m_t^2 - m_b^2)(y_1^2 + y_2^2) - 4m_t m_b y_1 y_2], \quad (4)$$

where $\kappa = \kappa(m_{H^+}^2, m_t^2, m_b^2)$, $\kappa(x, y, z) \equiv ((x - y - z)^2 - 4yz)^{1/2}$, and $N_C = 3$.

The vertex corrections due to gluon and gluino exchanges at the vertex (Fig. 1b), $\delta Y_i^{(v,g)}$ and $\delta Y_i^{(v,\tilde{g})}$, respectively, are given by:

$$\begin{aligned} \delta(Y_1 P_R + Y_2 P_L)^{(v,g)} &= \frac{\alpha_s C_F}{4\pi} \left\{ 2[B_0(m_t^2, 0, m_t^2) + B_0(m_b^2, 0, m_b^2) - r \right. \\ &\quad \left. - (m_{H^+}^2 - m_t^2 - m_b^2)C_0(\lambda^2, m_t^2, m_b^2)](y_1 P_R + y_2 P_L) \right. \\ &\quad \left. - 2m_t C_1(\lambda^2, m_t^2, m_b^2)[(m_t y_1 + m_b y_2)P_R + (m_t y_2 + m_b y_1)P_L] \right. \\ &\quad \left. - 2m_b C_2(\lambda^2, m_t^2, m_b^2)[(m_t y_2 + m_b y_1)P_R + (m_t y_1 + m_b y_2)P_L] \right\}, \\ \delta(Y_1 P_R + Y_2 P_L)^{(v,\tilde{g})} &= \frac{\alpha_s C_F}{4\pi} \left\{ 2G_{ij} \left[-m_{\tilde{g}} C_0(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LR})_{ij} P_R + (\alpha_{RL})_{ij} P_L\} \right. \right. \\ &\quad \left. + m_t C_1(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LL})_{ij} P_L + (\alpha_{RR})_{ij} P_R\} \right. \\ &\quad \left. + m_b C_2(m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \{(\alpha_{LL})_{ij} P_R + (\alpha_{RR})_{ij} P_L\} \right] \right\}. \end{aligned} \quad (5)$$

with $C_F = 4/3$ and

$$\begin{aligned} \alpha_{LL} &= \begin{pmatrix} \cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & -\cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \\ -\sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \end{pmatrix}, \quad \alpha_{LR} = \begin{pmatrix} -\cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & -\cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \\ \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \end{pmatrix}, \\ \alpha_{RL} &= \begin{pmatrix} -\sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \\ -\cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \end{pmatrix}, \quad \alpha_{RR} = \begin{pmatrix} \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \\ \cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \end{pmatrix}. \end{aligned} \quad (6)$$

G_{ij} are the tree-level couplings of H^+ to $\tilde{t}_i \tilde{b}_j$ ($i, j = 1, 2$) reading:

$$G_{ij} = \frac{g}{\sqrt{2}m_W} R^{\tilde{t}} \begin{pmatrix} m_b^2 \tan \beta + m_t^2 \cot \beta - m_W^2 \sin 2\beta & m_b(A_b \tan \beta + \mu) \\ m_t(A_t \cot \beta + \mu) & 2m_t m_b / \sin 2\beta \end{pmatrix} (R^{\tilde{b}})^{\dagger}. \quad (7)$$

Here $R^{\tilde{q}}$ ($\tilde{q} = \tilde{t}$ or \tilde{b}) is the \tilde{q} -mixing matrix

$$R_{i\alpha}^{\tilde{q}} = \begin{pmatrix} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\ -\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix} \quad (i = 1, 2; \alpha = L, R) \quad (8)$$

relating the squark states \tilde{q}_L and \tilde{q}_R to the mass-eigenstates \tilde{q}_1 and \tilde{q}_2 ($m_{\tilde{q}_1} < m_{\tilde{q}_2}$): $\tilde{q}_i = R_{i\alpha}^{\tilde{q}} \tilde{q}_\alpha$. $R^{\tilde{q}}$ diagonalizes the squark mass matrix [3]:

$$\begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{RL}^2 & m_{RR}^2 \end{pmatrix} = (R^{\tilde{q}})^\dagger \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix} R^{\tilde{q}}, \quad (9)$$

where

$$m_{LL}^2 = M_{\tilde{Q}}^2 + m_q^2 + m_Z^2 \cos 2\beta (I_q - Q_q \sin^2 \theta_W), \quad (10)$$

$$m_{RR}^2 = M_{\{\tilde{U}, \tilde{D}\}}^2 + m_q^2 + m_Z^2 \cos 2\beta Q_q \sin^2 \theta_W, \quad (11)$$

$$m_{LR}^2 = m_{RL}^2 = \begin{cases} m_t(A_t - \mu \cot \beta) & (\tilde{q} = \tilde{t}) \\ m_b(A_b - \mu \tan \beta) & (\tilde{q} = \tilde{b}) \end{cases}. \quad (12)$$

As usually, we introduce a gluon mass λ for the regularization of the infrared divergence. Here we define the functions B_0 , B_1 , C_0 , C_1 , and C_2 as in [11, 12]:

$$\begin{aligned} [B_0, k^\mu B_1](k^2, m_0^2, m_1^2) &= \int \frac{d^D q}{i\pi^2} \frac{[1, q^\mu]}{(q^2 - m_0^2)((q+k)^2 - m_1^2)} \\ [C_0, k_t^\mu C_1 - k_{\bar{b}}^\mu C_2](m_0^2, m_1^2, m_2^2) &= \int \frac{d^D q}{i\pi^2} \frac{[1, q^\mu]}{(q^2 - m_0^2)((q+k_t)^2 - m_1^2)((q-k_{\bar{b}})^2 - m_2^2)}. \end{aligned} \quad (13)$$

Here k_t and $k_{\bar{b}}$ are the external momenta of t and \bar{b} , respectively. The parameter r in eq. (5) and following equations shows the dependence on the regularization: $r = 1$ for dimensional regularization and $r = 0$ for the dimensional reduction (DR) [13]. The dependence on r , however, disappears in our final result.

Now we turn to the quark wave-function renormalization due to the graphs of Fig. 1c. The two-point vertex function for $\bar{q}q$ can be written as:

$$\not{k}(1 + \Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R) - (m_q + \Sigma_L^q(k^2)P_L + \Sigma_R^q(k^2)P_R). \quad (14)$$

Here we have $\Sigma_L^q(k^2) = \Sigma_R^q(k^2) \equiv \Sigma^q(k^2)$. The correction to the amplitude from the wave-function renormalization has the form:

$$\begin{aligned} \delta(Y_1 P_R + Y_2 P_L)^{(w)} &= -\frac{1}{2}(\Pi_L^t(m_t^2) + \Pi_R^b(m_b^2))y_1 P_R - \frac{1}{2}(\Pi_R^t(m_t^2) + \Pi_L^b(m_b^2))y_2 P_L \\ &\quad + (m_t \dot{\Sigma}^t(m_t^2) - m_t^2 \dot{\Pi}^t(m_t^2) + m_b \dot{\Sigma}^b(m_b^2) - m_b^2 \dot{\Pi}^b(m_b^2))(y_1 P_R + y_2 P_L), \end{aligned} \quad (15)$$

with $\dot{\Pi}^q \equiv \frac{1}{2}(\dot{\Pi}_L^q + \dot{\Pi}_R^q)$ and $\dot{X} \equiv \frac{dX}{dk^2}$. The explicit calculation yields:

$$(\Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R)^{(g)} = \frac{\alpha_s C_F}{4\pi} [-2B_1(k^2, m_q^2, \lambda^2) - r], \quad (16)$$

$$\begin{aligned} (\Pi_L^q(k^2)P_L + \Pi_R^q(k^2)P_R)^{(\tilde{g})} &= -\frac{\alpha_s C_F}{4\pi} \left[2(\cos^2 \theta_{\tilde{q}} P_L + \sin^2 \theta_{\tilde{q}} P_R) B_1(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) \right. \\ &\quad \left. + 2(\sin^2 \theta_{\tilde{q}} P_L + \cos^2 \theta_{\tilde{q}} P_R) B_1(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2) \right], \end{aligned}$$

$$\Sigma^q(k^2)^{(g)} = \frac{\alpha_s C_F}{4\pi} m_q [4B_0(k^2, m_q^2, \lambda^2) - 2r], \quad (17)$$

$$\Sigma^q(k^2)^{(\tilde{g})} = \frac{\alpha_s C_F}{4\pi} [m_{\tilde{g}} \sin 2\theta_{\tilde{q}} (B_0(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) - B_0(k^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2))]. \quad (18)$$

Finally, there are additional corrections $\delta Y_i^{(0)}$ by the renormalization of the quark masses in the couplings of eq. (3) (In the $\overline{\text{DR}}$ scheme equivalent corrections are necessary as one takes the physical masses of the quarks as input):

$$\begin{aligned} \delta Y_1^{(0)} = \delta y_1 &= \frac{g}{\sqrt{2}m_W} \delta m_b \tan \beta, \\ \delta Y_2^{(0)} = \delta y_2 &= \frac{g}{\sqrt{2}m_W} \delta m_t \cot \beta, \end{aligned} \quad (19)$$

$$\begin{aligned} \text{with } \delta m_q &= \delta m_q^{(g)} + \delta m_q^{(\tilde{g})}, \\ \delta m_q^{(g)} &= -\frac{\alpha_s C_F}{4\pi} [2m_q (B_0(m_q^2, 0, m_q^2) - B_1(m_q^2, 0, m_q^2) - \frac{r}{2})], \text{ and} \\ \delta m_q^{(\tilde{g})} &= -\frac{\alpha_s C_F}{4\pi} [\sin 2\theta_{\tilde{q}} m_{\tilde{g}} (B_0(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) - B_0(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2)) \\ &\quad + m_q (B_1(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_1}^2) + B_1(m_q^2, m_{\tilde{g}}^2, m_{\tilde{q}_2}^2))] . \end{aligned} \quad (20)$$

Taking all contributions eqs. (3, 5, 15, 19) together, we get the one-loop corrected couplings $Y_i = y_i + \delta Y_i = y_i + \delta Y_i^{(0)} + \delta Y_i^{(v)} + \delta Y_i^{(w)}$ with contributions to $\delta Y_i^{(0),(v),(w)}$ from gluon and gluino exchanges (see eq. (2)). It can be readily seen that they are ultraviolet finite but still infrared divergent. The one-loop corrected decay width to $\mathcal{O}(\alpha_s)$ is then given by

$$\begin{aligned} \Gamma(H^+ \rightarrow t\bar{b}) &= \frac{N_C \kappa}{16\pi m_{H^+}^3} \left[(m_{H^+}^2 - m_t^2 - m_b^2) (y_1^2 + y_2^2 + 2y_1 \text{Re}(\delta Y_1) + 2y_2 \text{Re}(\delta Y_2)) \right. \\ &\quad \left. - 4m_t m_b (y_1 y_2 + y_1 \text{Re}(\delta Y_2) + y_2 \text{Re}(\delta Y_1)) \right]. \end{aligned} \quad (21)$$

3 Inclusion of the gluon emission

For the cancellation of the infrared divergencies ($\lambda \rightarrow 0$) it is necessary to include the $\mathcal{O}(\alpha_s)$ contribution from real gluon emission as shown in Fig. 1d.

The decay width of $H^+ \rightarrow t + \bar{b} + g$ is given by

$$\Gamma(H^+ \rightarrow t\bar{b}g) = \frac{\alpha_s C_F N_C}{4\pi^2 m_{H^+}} \left[(y_1^2 + y_2^2) \{ J_1 - (m_{H^+}^2 - m_t^2 - m_b^2) J_2 \right. \\ \left. + (m_{H^+}^2 - m_t^2 - m_b^2)^2 I_{12} \} + 4m_t m_b y_1 y_2 \{ J_2 - (m_{H^+}^2 - m_t^2 - m_b^2) I_{12} \} \right], \quad (22)$$

with the integrals

$$I_{12} = \frac{1}{4m_{H^+}^2} \left[-2 \ln \left(\frac{\lambda m_{H^+} m_t m_b}{\kappa^2} \right) \ln \beta_0 + 2 \ln^2 \beta_0 - \ln^2 \beta_1 - \ln^2 \beta_2 \right. \\ \left. + 2 \text{Sp}(1 - \beta_0^2) - \text{Sp}(1 - \beta_1^2) - \text{Sp}(1 - \beta_2^2) \right] \quad (23)$$

$$J_1 = \frac{1}{2} I_1^2 + \frac{1}{2} I_2^2 + I = -\frac{1}{2} I_1^0 - \frac{1}{2} I_2^0 \\ = \frac{1}{8m_{H^+}^2} \left[(\kappa^2 + 6m_t^2 m_b^2) \ln \beta_0 - \frac{3}{2} \kappa (m_{H^+}^2 - m_t^2 - m_b^2) \right] \quad (24)$$

$$J_2 = m_t^2 I_{11} + m_b^2 I_{22} + I_1 + I_2 \\ = -\frac{1}{4m_{H^+}^2} \left[2\kappa \ln \left(\frac{\lambda m_{H^+} m_t m_b}{\kappa^2} \right) + 4\kappa + (m_{H^+}^2 + m_t^2 + m_b^2) \ln \beta_0 \right. \\ \left. + 2m_t^2 \ln \beta_1 + 2m_b^2 \ln \beta_2 \right]. \quad (25)$$

Here

$$\beta_0 \equiv \frac{m_{H^+}^2 - m_t^2 - m_b^2 + \kappa}{2m_t m_b}, \quad \beta_1 \equiv \frac{m_{H^+}^2 - m_t^2 + m_b^2 - \kappa}{2m_{H^+} m_b}, \\ \beta_2 \equiv \frac{m_{H^+}^2 + m_t^2 - m_b^2 - \kappa}{2m_{H^+} m_t}, \quad \text{Sp}(x) = -\int_0^x \frac{dt}{t} \ln(1-t), \quad (26)$$

and $\kappa = \kappa(m_{H^+}^2, m_t^2, m_b^2)$. The definitions and the explicit forms of the I 's are given in [11].

The one-loop corrected decay width to $\mathcal{O}(\alpha_s)$ including the real gluon emission can be written as:

$$\Gamma^{\text{corr}}(H^+ \rightarrow t\bar{b} + t\bar{b}g) \equiv \Gamma(H^+ \rightarrow t\bar{b}) + \Gamma(H^+ \rightarrow t\bar{b}g) \\ = \Gamma^{\text{tree}}(H^+ \rightarrow t\bar{b}) + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino}), \quad (27)$$

with Γ^{tree} given by eq. (4), and

$$\begin{aligned} \delta\Gamma(\text{gluon}) = & \frac{N_C \kappa}{16\pi m_{H^+}^3} \left[2(m_{H^+}^2 - m_t^2 - m_b^2) \left(y_1 \text{Re}(\delta Y_1^{(g)}) + y_2 \text{Re}(\delta Y_2^{(g)}) \right) \right. \\ & \left. - 4m_t m_b \left(y_1 \text{Re}(\delta Y_2^{(g)}) + y_2 \text{Re}(\delta Y_1^{(g)}) \right) \right] + \Gamma(H^+ \rightarrow t\bar{b}g), \end{aligned} \quad (28)$$

$$\begin{aligned} \delta\Gamma(\text{gluino}) = & \frac{N_C \kappa}{16\pi m_{H^+}^3} \left[2(m_{H^+}^2 - m_t^2 - m_b^2) \left(y_1 \text{Re}(\delta Y_1^{(\tilde{g})}) + y_2 \text{Re}(\delta Y_2^{(\tilde{g})}) \right) \right. \\ & \left. - 4m_t m_b \left(y_1 \text{Re}(\delta Y_2^{(\tilde{g})}) + y_2 \text{Re}(\delta Y_1^{(\tilde{g})}) \right) \right]. \end{aligned} \quad (29)$$

We have checked that the corrected width of eq. (27) is infrared finite.

4 Numerical Results and Discussion

We now turn to the numerical evaluation of the corrected width eq.(27). As the standard QCD corrections have already been calculated [7], it is interesting here to study the influence of the gluino (and \tilde{t}_i, \tilde{b}_j) exchange corrections $\delta\Gamma(\text{gluino})$. The whole analysis depends on the following parameters defined at the weak scale: $m_{H^+}, \tan\beta, \mu, A_t, A_b, M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}$, and $m_{\tilde{g}}$. For simplicity we assume $M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}}$ and $A_t = A_b \equiv A$. We have found that our final results are rather insensitive to these assumptions. We take $m_t = 180$ GeV, $m_b = 5$ GeV, $m_W = 80$ GeV, $m_Z = 91.2$ GeV, $\sin^2\theta_W = 0.23$, $g^2/(4\pi) = \alpha_2 = \alpha/\sin^2\theta_W = 0.0337$ and $\alpha_s = \alpha_s(m_{H^+})$. We use $\alpha_s(Q) = 12\pi/\{(33 - 2n_f)\ln(Q^2/\Lambda_{n_f}^2)\}$ with $\alpha_s(m_Z) = 0.12$ and the number of quark flavors $n_f = 5(6)$ for $m_b < Q \leq m_t$ (for $Q > m_t$).

In Fig. 2 we show the dependence of $\delta\Gamma(\text{gluino})$ as a function of A and $M_{\tilde{Q}}$ for $\tan\beta = 2$ (a) and 12 (b), and $(m_{H^+}, m_{\tilde{g}}, \mu) = (400, 550, 300)$ (GeV). We see that the size of the SUSY-QCD correction $\delta\Gamma(\text{gluino})$ can be large going up to $\sim 50\%$ and that it can be comparable to or even larger than the standard QCD correction $\delta\Gamma(\text{gluon})$ in a large parameter region. For fixed $\tan\beta$, $\delta\Gamma(\text{gluino})$ has a strong dependence on the parameters $M_{\tilde{Q}}$ and A which determine the masses and couplings of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$. $\delta\Gamma(\text{gluino})$ is smaller for larger masses of \tilde{t}_1 and \tilde{b}_1 : for $\tan\beta = 2$, the correction due to $\delta\Gamma(\text{gluino})$ is about -15% for $(M_{\tilde{Q}}, A) = (100 \text{ GeV}, 300 \text{ GeV})$

(where $m_{\tilde{t}_1} \simeq 119$ GeV, and $m_{\tilde{b}_1} \simeq 98$ GeV), but it is still $\sim -5\%$ for larger squark masses $(M_{\tilde{Q}}, A) = (400 \text{ GeV}, 300 \text{ GeV})$ (where $m_{\tilde{t}_1} \simeq 405$ GeV, and $m_{\tilde{b}_1} \simeq 399$ GeV). This tendency is consistent with the decoupling theorem for the MSSM. Notice also the different behaviour for $\tan\beta = 2$ and $\tan\beta = 12$.

In Fig. 3 we show the m_{H^+} dependence of Γ^{tree} , $\Gamma^{\text{tree}} + \delta\Gamma(\text{gluon})$, and $\Gamma^{\text{corr}} = \Gamma^{\text{tree}} + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino})$ for $\tan\beta = 2$ (a) and 12 (b), and $(m_{\tilde{g}}, \mu, M_{\tilde{Q}}, A) = (400, -300, 200, 200)$ (GeV). The parameter values correspond to fixed stop and sbottom masses: $m_{\tilde{t}_1} = 90$ GeV, $m_{\tilde{t}_2} = 366$ GeV, $m_{\tilde{b}_1} = 193$ GeV, and $m_{\tilde{b}_2} = 213$ GeV (for $\tan\beta = 2$) and $m_{\tilde{t}_1} = 173$ GeV, $m_{\tilde{t}_2} = 333$ GeV, $m_{\tilde{b}_1} = 152$ GeV, and $m_{\tilde{b}_2} = 247$ GeV (for $\tan\beta = 12$). (Note that for $m_{\tilde{g}} = 400$ GeV the D0 mass limit of the mass-degenerate squarks of five flavors (excluding $\tilde{t}_{1,2}$) is $m_{\tilde{q}} \gtrsim 140$ GeV [14].) We see again that the correction $\delta\Gamma(\text{gluino})$ can be quite large and that it is comparable to or even larger than $\delta\Gamma(\text{gluon})$ in a large region. Quite generally, the corrections $\delta\Gamma(\text{gluon})$ and $\delta\Gamma(\text{gluino})$ are bigger for larger $\tan\beta$, but it can happen that they partly cancel each other. The correction $\delta\Gamma(\text{gluon})$ has already been calculated in [7]. Our results on $\delta\Gamma(\text{gluon})$ agree numerically with ref. 7 within 10%.

In Fig. 4 we show a contour-plot for $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$ in the $\tan\beta - m_{\tilde{g}}$ plane for $(m_{H^+}, \mu, M_{\tilde{Q}}, A) = (400, -300, 250, 300)$ GeV. This correction rises with increasing $\tan\beta$, going up to 50%! Concerning the $m_{\tilde{g}}$ dependence, $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$ increases up to $m_{\tilde{g}} = 300 - 450$ GeV and then decreases gradually as $m_{\tilde{g}}$ increases. It is striking that even for a large gluino mass (~ 1 TeV) $\frac{\delta\Gamma(\text{gluino})}{\Gamma^{\text{corr}}}$ is larger than 10% for $\tan\beta \gtrsim 3$. From Figs. 2 and 4 we see that the correction $\delta\Gamma(\text{gluino})$ decreases much faster for increasing $M_{\tilde{Q}}$ than for increasing $m_{\tilde{g}}$.

In Fig. 5 we show contour lines of $\delta\Gamma(\text{gluino})$ in the $\mu - A$ plane for $\tan\beta = 2$ (a) and 12 (b), and $(m_{H^+}, m_{\tilde{g}}, M_{\tilde{Q}}) = (400, 550, 300)$ GeV. This correction has a strong dependence on μ and a significant dependence on A . We have found that the

A dependence for $\tan\beta = 1$ is much stronger than that for $\tan\beta = 2$.

The reason for the large contribution of $\delta\Gamma(\text{gluino})$ as compared to $\delta\Gamma(\text{gluon})$ is the following: The vertex-correction part of the gluino-exchange [gluon-exchange] corrections (see Fig. 1b and eq. (5)) is proportional to the $H^+\tilde{t}\tilde{b}$ coupling [$H^+\bar{t}b$ coupling] which is essentially $\sim (A_t + \mu \tan\beta)h_t \cos\beta + (A_b + \mu \cot\beta)h_b \sin\beta$ [$\sim h_t \cos\beta + h_b \sin\beta$]. Hence the vertex-correction part of the gluino-exchange corrections $\delta\Gamma(\text{gluino})$ can be strongly enhanced relative to that of the gluon-exchange corrections $\delta\Gamma(\text{gluon})$ in the case the \tilde{q} -mixing parameters A and μ are large. In this case \tilde{t}_1 and \tilde{b}_1 tend to be light due to a large mass-splitting. Note that the \tilde{b} -mixing effect plays a very important role for large $\tan\beta$.

5 Conclusion

Summarizing, we have performed a complete calculation of the $\mathcal{O}(\alpha_s)$ QCD corrections to the width of $H^+ \rightarrow t\bar{b}$ within the MSSM. We have found that the QCD corrections are quite important. A detailed numerical analysis has shown that the SUSY-QCD corrections (due to gluino, \tilde{t} and \tilde{b} exchanges) can be comparable to or even larger than the standard QCD corrections in a large region of the MSSM parameter space; here the mixings of $\tilde{t}_L - \tilde{t}_R$ and $\tilde{b}_L - \tilde{b}_R$ play a crucial role. This could significantly affect the phenomenology of the H^+ search.

After having finished this study, we have been informed on a recent paper [15] dealing with the same subject.

Acknowledgements

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wissenschaftlichen Forschung” of Austria, project no. P10843-PHY.

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Figure Captions

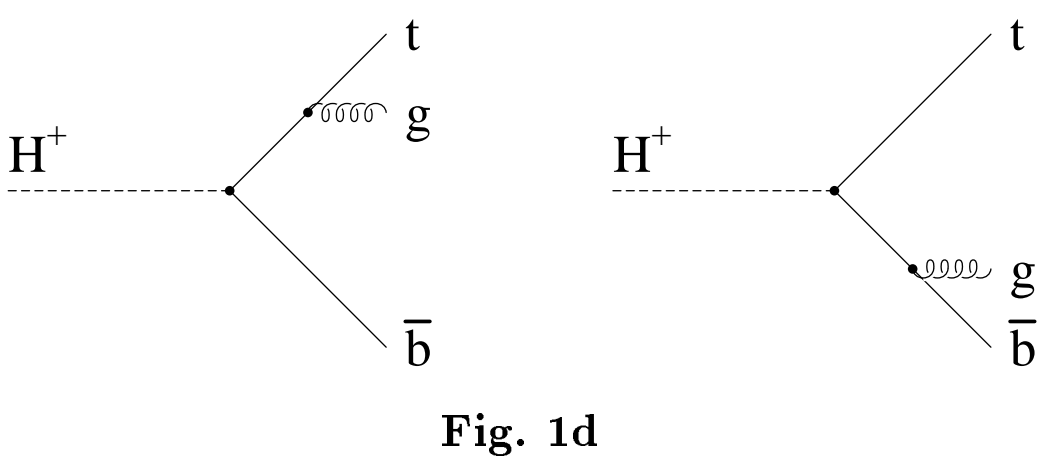
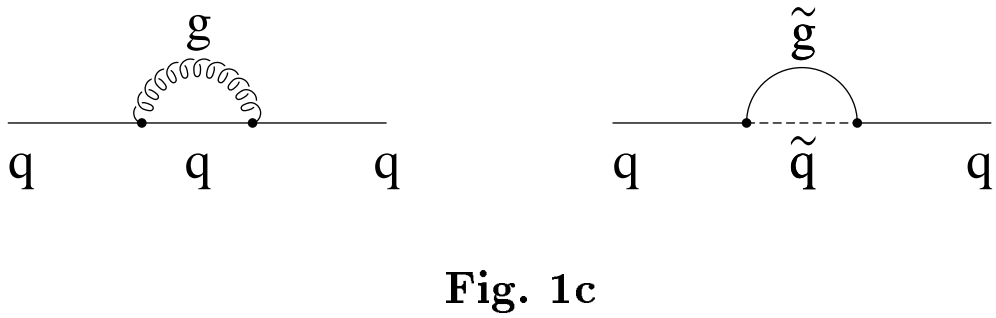
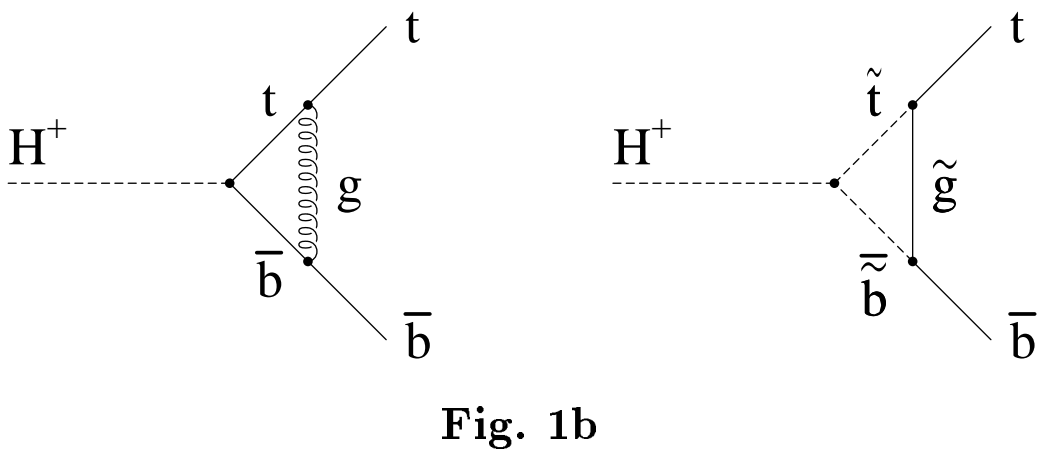
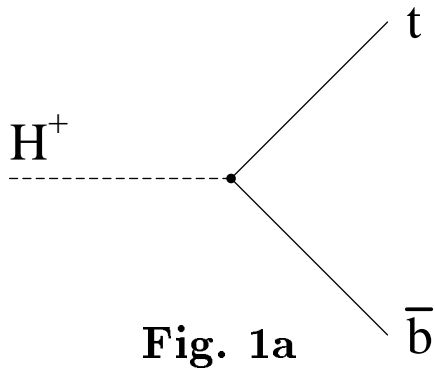
Fig. 1 All diagrams relevant for the calculation of the $\mathcal{O}(\alpha_s)$ QCD corrections to the width of $H^+ \rightarrow t\bar{b}$ in the MSSM.

Fig. 2 Contour lines of $\delta\Gamma(\text{gluino})$ (GeV) in the $A - M_{\tilde{Q}}$ plane for $\tan\beta = 2$ (a) and 12 (b), and $(m_{H^+}, m_{\tilde{g}}, \mu) = (400, 550, 300)$ (GeV). For these parameter values one has $(\Gamma^{\text{tree}} \text{ (GeV)}, \delta\Gamma(\text{gluon}) \text{ (GeV)}) = (4.10, 0.31)$ and $(1.91, -0.66)$ for Figs. 2a and 2b, respectively. The shaded area is excluded by the LEP bounds $m_{\tilde{t}_1, \tilde{b}_1} \gtrsim 45$ GeV. Note that for $m_{\tilde{g}} \simeq 550$ GeV one has no squark mass bound from D0 experiment [14].

Fig. 3 m_{H^+} dependence of Γ^{tree} (dashed line), $\Gamma^{\text{tree}} + \delta\Gamma(\text{gluon})$ (dot-dashed line), and $\Gamma^{\text{corr}} = \Gamma^{\text{tree}} + \delta\Gamma(\text{gluon}) + \delta\Gamma(\text{gluino})$ (solid line) for $\tan\beta = 2$ (a) and 12 (b), and $(m_{\tilde{g}}, \mu, M_{\tilde{Q}}, A) = (400, -300, 200, 200)$ (GeV).

Fig. 4 Contour lines of $\delta\Gamma(\text{gluino})/\Gamma^{\text{corr}}$ in the $\tan\beta - m_{\tilde{g}}$ plane for $(m_{H^+}, \mu, M_{\tilde{Q}}, A) = (400, -300, 250, 300)$ (GeV). The area below the dotted line is excluded by the LEP limit $m_{\tilde{\chi}_1^+} \gtrsim 45$ GeV (assuming $m_{\tilde{g}} = (\alpha_s/\alpha_2)M_2 \simeq 3.56M_2$), where $\alpha_2 = g^2/(4\pi)$, M_2 is the SU(2) gaugino mass, and $m_{\tilde{\chi}_1^+}$ is the lighter chargino mass.

Fig. 5 Contour lines of $\delta\Gamma(\text{gluino})$ (GeV) in the $\mu - A$ plane for $\tan\beta = 2$ (a) and 12 (b), and $(m_{H^+}, m_{\tilde{g}}, M_{\tilde{Q}}) = (400, 550, 300)$ GeV. For these parameter values one has $(\Gamma^{\text{tree}} \text{ (GeV)}, \delta\Gamma(\text{gluon}) \text{ (GeV)}) = (4.10, 0.31)$ and $(1.91, -0.66)$ for Figs. 5a and 5b, respectively. The shaded area is excluded by the LEP limits $m_{\tilde{t}_1, \tilde{b}_1, \tilde{\chi}_1^+} \gtrsim 45$ GeV. For $m_{\tilde{g}} \simeq 550$ GeV one has no $m_{\tilde{q}}$ limit from the D0 experiment [14].



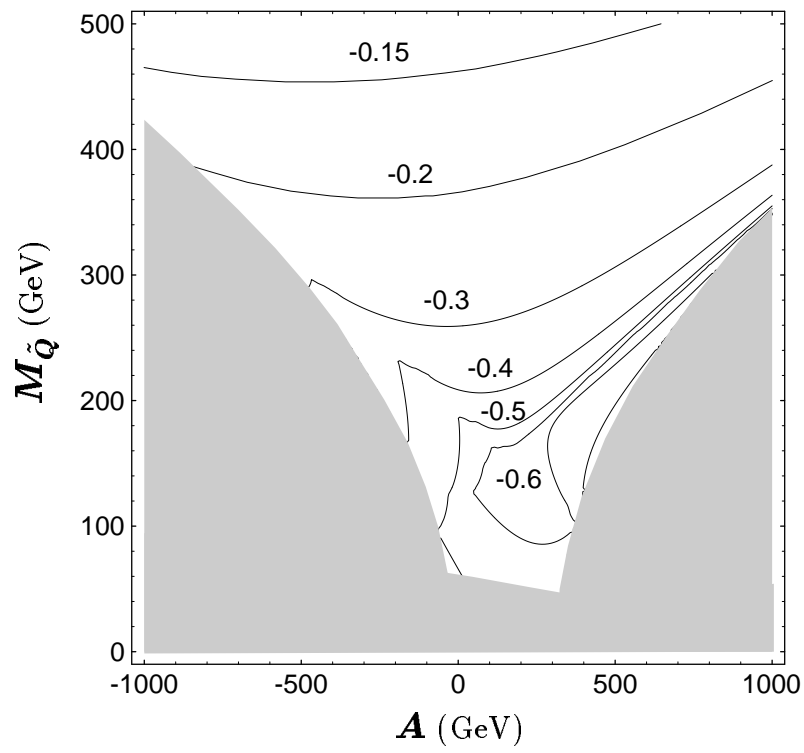


Fig. 2a

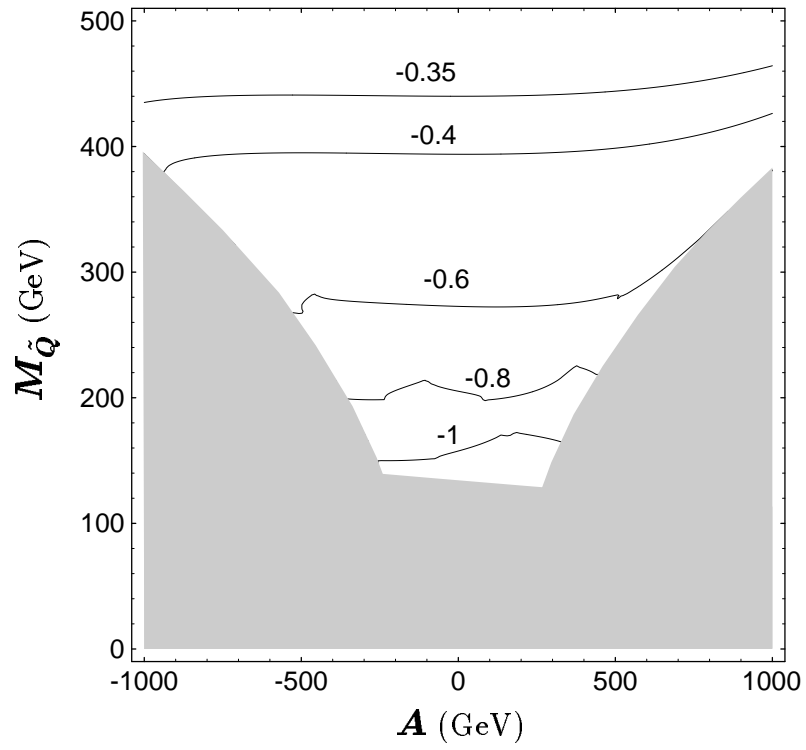


Fig. 2b

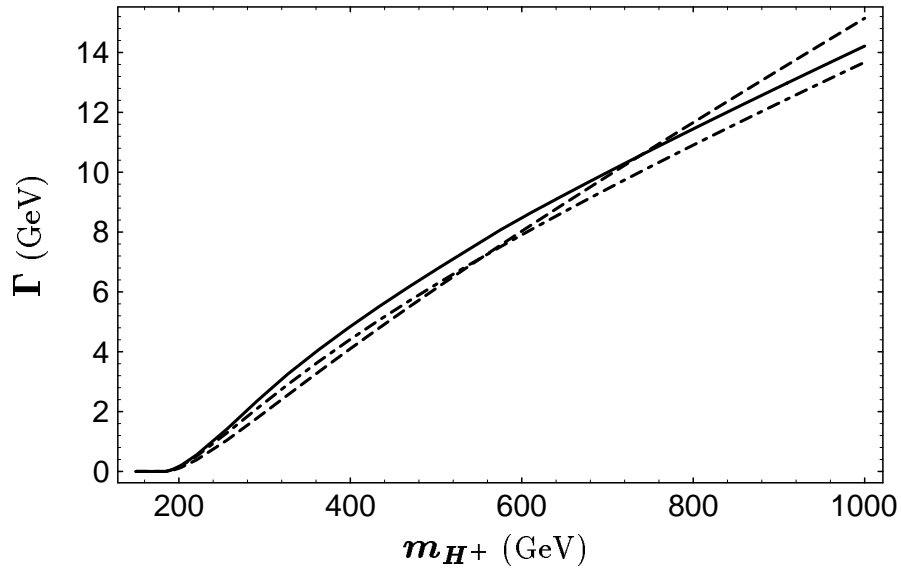


Fig. 3a

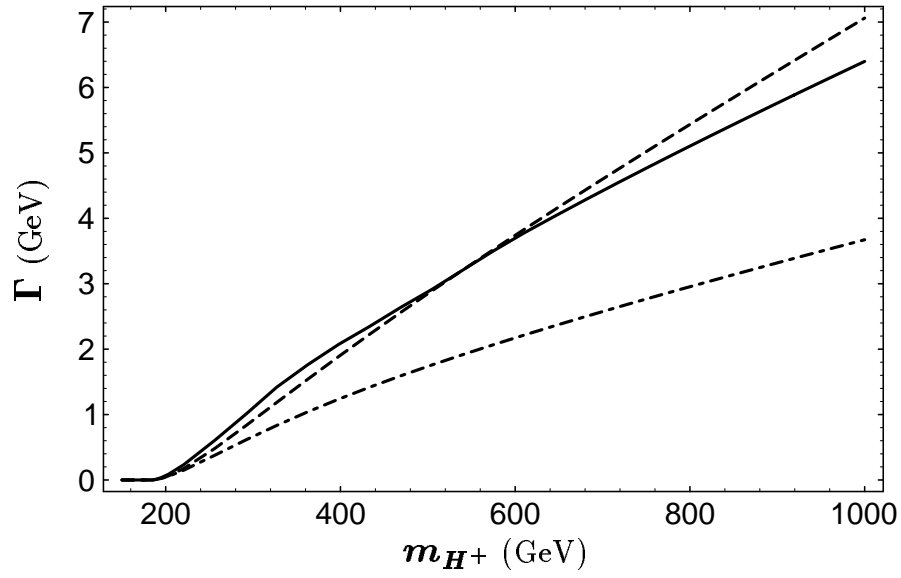


Fig. 3b

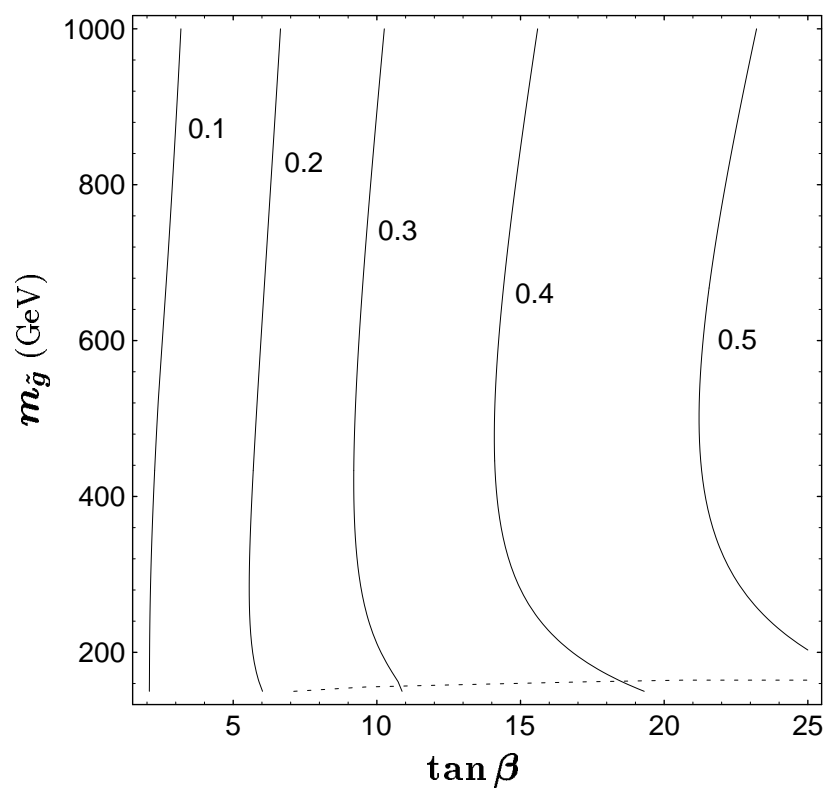


Fig. 4

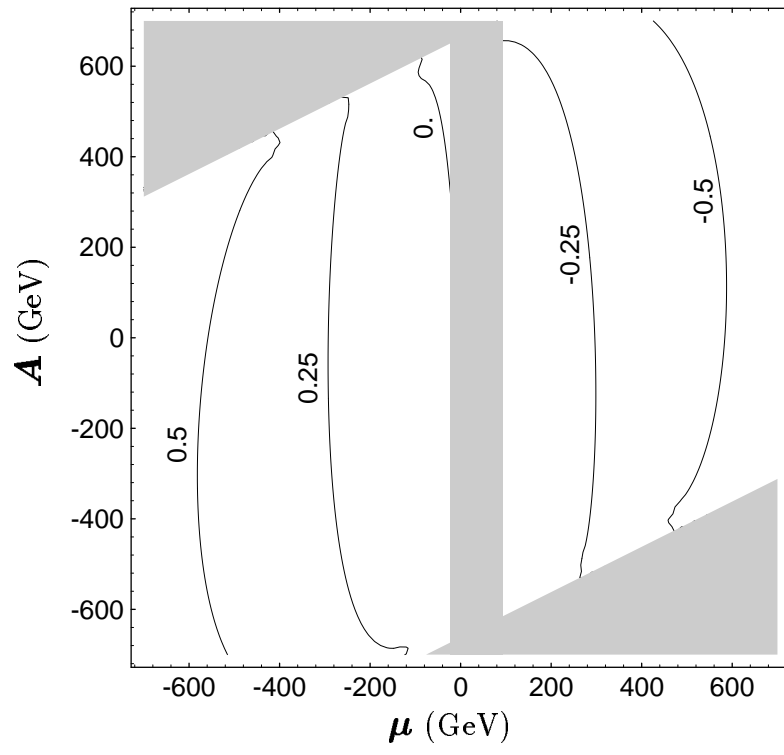


Fig. 5a

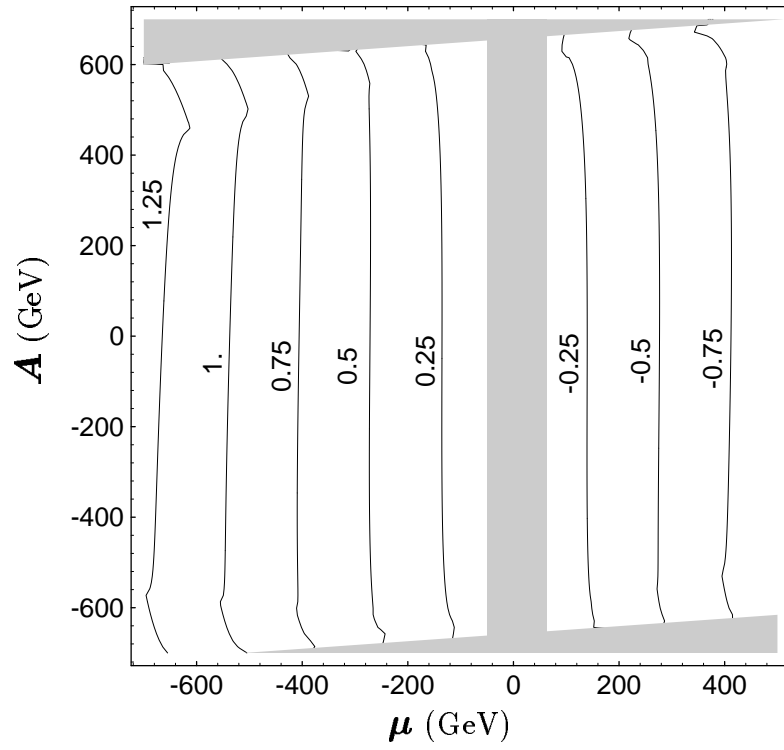


Fig. 5b